



Energy-efficient fuzzy model-based multivariable predictive control of a HVAC system



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ABSTRACT

In this paper the novel approach of a fuzzy model-based multivariable predictive functional control (FMBMPC) of a heating ventilating and air conditioning (HVAC) system is presented, which is implemented on a real-world test plant. The control law is derived in the state-space domain and is given in an analytical form without an optimization algorithm. The basic principles of the predictive control were extended in a fuzzy multivariable manner and the suggested tuning rules for the proposed control algorithm were depicted, which normally gives satisfactory results. The proposed approach introduces a compact and relatively simple design in the case of higher-order and nonminimal phase plants, but it is limited to open-loop stable plants. For the comparison a classical optimal proportional–integral (PI) controller was also designed and applied. The results show that the FMBMPC approach performs better due to the HVACs' nonlinear dynamics. In case of interactions influence rejection by the HVAC system, the FMBMPC algorithm outperforms the classical PI approach. The results also show that the proposed approach exhibits better reference-model tracking across a wider operating range. The energy consumption comparison shows that the FMBMPC approach is also more energy-efficient. A shortened literature review of applications of energy-efficient and MPC control for HVAC systems is also presented. FMBMPC control is interesting in the case of batch reactors, furnaces, pressure vessels, HVAC systems and any processes that have strong nonlinear dynamics, multivariable natures and long transport delays.

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1. Introduction

Industrial heating ventilating and air conditioning (HVAC) systems are widely used to condition the air (temperature, humidity, and pressure) in buildings, halls, and rooms to desired values. Advanced control strategies for HVAC systems have been extensively researched [1,2], but standard on/off and proportional–integral–(derivative) (PI(D)) controllers remain as the solution of first choice [3–7], resulting in inconsistent performance among such systems. With advances in control principles it is now feasible to implement proper control approach to overcome the inherent issues in HVAC control. In a past few years, papers

proposing modern control principles for HVAC systems, like optimal model predictive control (MPC) [8], H_∞ control [9], feedback linearization and backstepping [10], and feedback linearization in combination with MPC [11,12] were published.

The application of advanced process control technologies can provide the potential for significant energy savings. The development and implementation of effective control techniques for HVAC systems is very important. In recent years, classic control algorithms have been replaced with innovative, highly efficient and energy-saving solutions, because the design and implementation of more complex control techniques have become feasible. New methods help to attain the maximum energy efficiency, while optimally fulfilling the control and convenience demands, and also prolonging the lifetime of systems and extending maintenance cycles. The goal of making buildings more energy efficient represents an area for the application of innovative concepts and technologies on the level of buildings and devices. With the application of bioclimatic principles and the appropriate regulation of building processes, high standard indoor-air conditions can be met

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Nomenclature

$\bar{\mathbf{A}}_m$	frozen-time system matrix
$\mathbf{A}_{m,j}$	system matrix
\mathbf{A}_r	reference model system matrix
a_{r1}	first discrete time constant of the reference model
a_{r2}	second discrete time constant of the reference model
a_{rm}	last discrete time constant of the reference model
$\bar{\mathbf{B}}_m$	frozen-time input matrix
$\mathbf{B}_{m,j}$	input matrix
\mathbf{B}_r	reference model input matrix
$\bar{\mathbf{C}}_m$	frozen-time output matrix
$\mathbf{C}_{m,j}$	output matrix
\mathbf{C}_r	reference model output matrix
c_w	water specific heat capacity
d_1	hot water supply temperature for the heating coil hydraulic circuit
d_2	cool water supply temperature for the cooling coil hydraulic circuit
d_3	air temperature at the inlet of cooling coil 2
d_4	air relative humidity at the inlet of cooling coil 2
D_{mc}	time delay for cooler to temperature
D_{mh}	time delay for heater to relative humidity
\mathbf{G}_0	invertible matrix by the FMBMPC control law
\mathbf{G}_R	matrix by the FMBMPC control law
H	coincidence horizon
H_r	subscript for the relative humidity
i	number of current premise element
\mathbf{I}	identity matrix
j	number of current fuzzy rule
$J(u,k)$	criterion function
k	time sample
$K_{p,Hr}$	proportional gain for the relative humidity
$K_{p,T}$	proportional gain for the temperature
l	number of steps for prediction
n	number of model states
N_1	minimum horizon
N_2	maximum horizon
n_f	number of all fuzzy rules
N_u	control horizon
P_c	consumed power
$\mathbf{P}_{j,q}$	fuzzy sets
q	number of premise elements
r	number of consequence elements
r_f	residual of the fuzzy model
\mathbf{R}_j	fuzzy rules
$\bar{\mathbf{R}}_m$	frozen-time residual vector
$\mathbf{R}_{m,j}$	residual vector
T	subscript for the temperature
t_c	operating time
T_{Dm}	time delay
$T_{i,Hr}$	reset time for the relative humidity
$T_{i,T}$	reset time for the temperature
T_{mc}	time constant for cooler to temperature
T_{mh}	time constant for heater to relative humidity
T_{mn}	continuous time constant of the model
T_{r1}	first reference model time constant
T_{r2}	second reference model time constant
T_m	continuous time constant of the reference model
T_s	sampling time
u	control signal
u_1	valve opening for the cooling coil 2
u_2	valve opening for the heating coil 2

u_m	system inputs
w	reference signal
W_c	consumed energy
x_c	consequence of the fuzzy system
x_{c1}	first consequence element
x_{cr}	last consequence element
x_m	system states
x_m^0	non-delayed system states
x_p	premise of the fuzzy system
x_{p1}	first premise element
x_{pi}	current premise element
x_{pq}	last premise element
x_r	reference model state vector
y_1	air temperature
y_2	air relative humidity
y_m	model output
y_m^0	non-delayed model output
y_p	process output
y_p^0	non-delayed process output
y_r	reference model output
$\beta_j(x_p)$	function
Δ_m	objective increment of the model output
Δ_p	objective increment of the process output
ϑ_{in}	inlet water temperature
ϑ_{out}	outlet water temperature
λ	weight for criterion function
$\mu_{p_j,i}(x_{pi})$	real function
ρ	relative degree of the model
$\phi(x_m, u_m, R_{m,j}, k)$	linear function
ϕ_{mf}	water mass flow

and a reduction in energy use can be achieved [13]. Energy-saving strategies have become a priority in energy policies in many countries due to a significant increase in the energy consumption in buildings, where the largest amounts of energy (around 40%) are used [14,15].

Some researchers have investigated the energy efficiency of HVAC systems with different control approaches, for example, the learning-based model predictive control of a HVAC system is presented in [16], where a large reduction in energy usage without any reduction in occupant comfort is achieved. The multivariable nonlinear adaptive control algorithms of temperature and humidity in HVAC systems were studied [17] to achieve optimal energy-saving control. HVAC system modelling and PID control according to energy efficiency and comfort criteria were performed [18], and stochastic MPC for building climate control that uses weather predictions was developed and analysed [19], which makes use of weather forecasts to compute how much energy and which low/high energy cost actuators are needed to maintain room temperature at the required comfort levels.

In [20] a review of the model predictive control of HVAC control systems is presented. The development of the MPC control approach for HVAC systems has intensified in recent years, due to advantages like the use of a system model for anticipatory control actions, rather than corrective control, the integration of a disturbance model for disturbance rejection, the ability to handle constraints and uncertainties, the ability to handle time-varying system dynamics and a wide range of operating conditions, and the ability to cope with slow-moving processes that have considerable time delays. Among the advanced control approaches, MPC is one of the most promising techniques because of its ability to integrate disturbance rejection, constraint handling, and slow-moving dynamic control and energy conservation strategies

into a controller formulation. The soft or intelligent control techniques reviewed in [2] include controllers based on artificial neural networks, fuzzy logic, and genetic algorithms. Intelligent control techniques such as neuro- and genetic-fuzzy approaches were reviewed in [21]. A review of fuzzy modelling and the control of HVAC systems were published in [22]. A review of hybrid controllers resulting from the fusion of hard and soft control techniques was also provided in [2]. Several MPC strategies have been applied to both single-zone and multi-zone buildings, for example, a single-story office building [23], a factory building [11], a small studio apartment [24], a large university building [25,26], a test room [27–29], a shed [30], and a multi-story office building [31].

Multi-input–multi-output (MIMO) MPC was used for the water flow valve in [32] to control the temperature of multiple zones, and MPC was also applied to regulate the evaporator temperature and pressure by controlling the electronic expansion valve and compressor speed. In [33] a MIMO controller to control the temperature and ventilation of multiple zones in a building with a MPC strategy is proposed. The authors in [34] presented a new family of recursive modelling MPC controllers for use with low-cost thermic controllable radiator valves, which are becoming an increasingly popular domestic technology. A key contribution of the paper is the ability of the presented control methodologies to maintain superior temperature regulation, despite the use of oversized heat emitters. The authors in [35] and [36] showed that MPC is well suited to the control of an indoor environment. An upgrade of the conventional MPC, the adaptive multiple model MPC, was then implemented for the optimization of thermal storage in buildings [37]. Simulations from [38] show that the controller with only outside weather and zone humidity measurements does not result in large savings, controllers with only occupancy measurements result in considerable energy savings, and the controllers that use occupancy measurements along with the measurements of zone humidity and outside weather, result in huge energy savings. The authors in [39] and [40] introduced new cost functions for the MPC approach, which minimizes the energy consumption and, hence, the cost of electricity for the user, while maintaining the thermal comfort in the building. In [41] a multi-structural fast nonlinear model predictive controller is presented. The methodology retains the advantages of linear classical MPC and solves the nonlinearity issues involved in thermal comfort and energy-conservation-oriented control. In [42] the authors proposed a framework that learns the building occupants' thermal comfort profiles using a fuzzy predictive model and controls the HVAC system using a complementary control strategy, which can be integrated with minimum intrusion. The framework was validated in an office building and considerable improvements in the occupants' satisfaction and energy use were achieved. The multiple local models approach is presented in [37], which achieves the desired performance. The Building Energy Model is used to construct local models for the adaptive multi-model MPC, and the evaluation results show that the adaptive MMPC approach outperforms the storage priority control. The MIMO controller for the indoor air temperature and relative humidity in a direct expansion air-conditioning system, based on the linearized dynamic model is presented in [43], and in [44] model predictive control strategies for buildings with mixed-mode cooling are presented and their potential performance bounds in terms of energy savings within the thermal comfort constraints are demonstrated. A series of MPC techniques has been explored for optimizing the control sequences for window operation in mixed-mode buildings and the results for a simplified mixed-mode office building have been presented in [45], where the results show the ability to save upwards of 40% of the cooling energy, even in existing facilities. In [46] the MPC control system was tested on two office buildings. The results presented from real-world trials demonstrated energy improvements of between 19% and 32% over existing control strategies.

For any MPC approach a model of the process is required. In the past many authors dealt with the HVAC system modelling and simulation, an overview is given in [47], where the categorization of tools for system design and analysis is introduced, approaches for modelling of HVAC components, control and systems in general are summarized, and an overview together with the usage suggestion of simulation solutions is presented. A simulator application developed in a combined Matlab/Simulink and Dymola/Modelica environment is presented in [48]. The simulator mirrors the functioning of the control system and the dynamics of the indoor environment, and can emulate the response of conventional ON/OFF controllers as well as fuzzy controllers. For example, a two-stage model was developed [49], which first learns the thermal patterns within the building when the system is OFF. This model is then included in the model when the system is ON, allowing a prediction of the effect of the system configuration more accurately. The model is scalable to similar systems and thus can be used to improve the efficiency of HVAC systems by helping to determine more effective control schemes.

When using HVAC systems we encounter, with rather high time constants, considerable transport delays, and actuating signals that are typically subject to constraints (e.g., valve position). The HVAC system control is also unique and challenging due to time-varying disturbances, poor data due to low resolution of analogue-to-digital converters, accuracy of sensors, lack of access to network forecasting and environmental information, and lack of supervisory control. Like most industrial plants, the HVAC system also exhibits a multivariable (MIMO) nature, which means that more than one variable has to be controlled. In many cases the controlled variables are coupled and the interactions are not negligible, which in our case means that if the air temperature increases the relative air humidity decreases. The HVAC system must be considered as truly multivariable, so some type of multivariable control has to be applied to the closed-loop system for the best performance. Classical multivariable process control is well known and was studied extensively in the past [50,51]. In addition, the nature of the HVAC system is inherently nonlinear, which implies the use of nonlinear control approaches. With a linear approach we encounter the problem of one operating point at which the closed-loop performance is good, and at the other operating points the performance is below the requirements. A nonlinear approach can overcome this problem and improve the closed-loop performance over the whole operating range. There are two main groups of approaches: the first group is based on nonlinear mathematical models and convex optimization [52], while the second group relies on an approximation of nonlinear process dynamics with nonlinear approximators such as Volterra and Wiener models, neural networks, piecewise-linear models, and in our case fuzzy models [53,54]. When using a (Takagi–Sugeno) T–S fuzzy model with model-based predictive control the choice of the fuzzy sets and the corresponding membership functions is always important when employing the Gustafson–Kessel clustering method [55] for the fuzzy identification.

The predictive control of industrial processes has become a very important area of research in recent years. In comparison with classical multivariable and nonlinear approaches the main advantage of fuzzy model-based multivariable predictive control (FMBMPC) is in its relatively simple design and the high-quality control performance. The fundamental methods that are essentially based on the principle of predictive control are dynamic matrix control, model algorithmic control, generalized predictive control, extended prediction self-adaptive control, extended horizon adaptive control, and predictive functional control, which are all developed for linear process models. The principle is based on the prediction of the process model output and the calculation of the control signal that brings the process output to the output of the reference

model (e.g., a reference trajectory). That is done in such a way as to minimize the difference between the desired reference and the output signal in a certain interval between two chosen prediction horizons, or in such a way as to minimize the mentioned difference in a certain horizon, which is called the coincidence horizon. The output signal of the control algorithm can be found by means of optimization or it can be calculated using the explicit control-law formula.

In this paper a new method of fuzzy model-based multivariable predictive control is presented that is based on the basic principles of predictive functional control (PFC), which are very easy to understand. This control strategy is well established in many industries, e.g., chemical, plastics, process control, and building automation [56–58], where the authors also designed the mathematical models of the industrial processes [59,60], and also in mobile robots trajectory tracking control [61] and in public transport systems [62,63]. The proposed control algorithm has wide application potential in different types of buildings and HVAC units as well as in a variety of other processes, for example, batch reactors, furnaces, pressure vessels. In fact, the algorithm can be used for the control of any processes that have strong nonlinear dynamics, multivariable natures, and long transport delays. However, the usage is not limited to the special control scenario presented in the paper; this is just one of many possible usage examples.

A simple predictive functional control algorithm [64] was first extended to a multivariable case (MPFC) [65] and to nonlinear systems (FPFC) [66]. Multivariable predictive approach has been studied, implemented and analyzed, also a new tuning strategy was developed [67]. For example, MIMO predictive approach was used for control of variable speed variable pitch wind turbines [68]. Fuzzy predictive approach is for example used for control of a solar power plant [69], and for supervisory control of gas turbines of combined cycle power plants [70]. The proposed FMBMPC approach is a combination of both multivariable and fuzzy model-based predictive control, on which some similar preliminary work was carried out in the past [71]. In recent years some similar approaches to fuzzy multivariable predictive control were presented, all of which were based on optimization algorithms. Generalized predictive control was adopted for the nonlinear multivariable system adaptive predictive control, for example, an application involving the control of a cement rotary kiln was based on the Alopex evolutionary optimization algorithm with a constrained T–S model [72], neural nonlinear model predictive control based on a direct computation of the gradient control vector during the predictive optimization task was studied as a more applicable approach of multivariable processes [73] and also applied to a multi-tank system based on a predictive optimization algorithm [74], and the multivariable fuzzy predictive control system of a nuclear power plant based on a fuzzy performance evaluation and a fuzzy decision optimization algorithm is presented [75]. The control law in the case of the FMBMPC approach is developed in a state-space domain and is given in explicit analytical form without an optimization algorithm, which makes the control algorithm also very easy to implement on programmable logic controllers (PLCs) and other hardware that is nowadays used in industrial practice. As the principles of PFC are the basis for the proposed control algorithm it easily copes with phase non-minimal and time-delayed dynamics. We must point out that the proposed approach was implemented on a real-world PLC and successfully tested on a real-world HVAC test plant with excellent performance.

The stability analysis of a class of MIMO fuzzy control systems was carried out using mostly used linear matrix inequality (LMI) approach [76], and in [77] another method using LaSalle's global invariant set theorem was presented, which reduces the complexity and conservativeness of the stability analysis.

This paper is organized as follows. Section 2 gives a general description of the HVAC system and presents the fuzzy identification of the process model. Section 3 deals with the concept of the fuzzy model-based multivariable predictive control and presents the controller tuning rules for the proposed algorithm. In Section 4 the implementation of the proposed control algorithm and the classical PI control are realized. In Section 5 the real-world test plant results are presented, discussed and the two algorithms are compared in terms of efficiency, accuracy and energy consumption. Finally, the conclusions are drawn in Section 6.

2. Plant and model

2.1. General description

Heating, ventilating, and air-conditioning systems are used to condition the air's temperature, humidity, and pressure. Conditioned air is needed in space-conditioning applications, e.g., in office buildings, and in industrial applications like the inlet air conditioning of combustion-engine test benches. The former task has to guarantee thermal comfort for people while ensuring energy-efficient operation. Control accuracy is of minor interest here. In contrast, the latter field of application imposes demanding specifications concerning control accuracy. We considered both the aspects, control accuracy and energy-efficient operation. The core components of the air-handling units are similar for both application areas. In the following, a HVAC test plant dedicated for research purposes will be described.

The test plant is capable of conditioning the dry-bulb air temperature (hereinafter referred to as the air temperature only) and relative air humidity and was designed to perform real-world control experiments. It consists of standard industrial components; hence the obtained results are closely related to the above-mentioned industrial systems. Fig. 1 shows a picture of the plant. The fundamental components of the plant are a fan, the heating and cooling coils and a steam humidifier. The heating coils are used to increase the air temperature, while the cooling coils are required to decrease the air temperature and to dehumidify air. The relative air humidity can be increased using the electrically actuated steam humidifier. Next to these devices, air dampers and filters, as well as measurement equipment, are installed. The operating mode of the test plant is set via the air dampers and the choice of the actuators. In the problem set-up given in the following, only a subset (shaded in Fig. 1) of the available actuators will be used.

The conditioned air is transported to a neighbouring factory hall, i.e., in Fig. 1, the lower damper on the left-hand side is closed, whereas the upper damper on the left-hand side is open. The controlled variables are the air temperature and the air relative humidity in the supply air duct to the factory hall, y_1 in K (or in °C, where y_1 in °C = y_1 in K – 273.15) and y_2 in % relative humidity, respectively. As actuators, the cooling coil 2 (u_1) and the heating coil 2 (u_2) are used. The cooling and heating power of the coils is adjusted with the help of valves. In the case of the cooling coil, the mass flow of cool water is adjusted, whereas in the case of the heating coil, the water inlet temperature of the fluid is adjusted by mixing with the cold return water. The hot and cool water supply temperatures for the heating and cooling coil hydraulic circuits are considered as the measurable disturbances d_1 and d_2 , respectively. The air temperature d_3 and the relative air humidity d_4 at the inlet of cooling coil 2 are measurable disturbances as well. The mentioned signals are depicted in Fig. 1. In addition, the water loops of the heating and cooling coil 2 are presented in Fig. 2. All the figures that present the temperature show the temperature units in °C (and not in K) due to it being a more natural presentation.

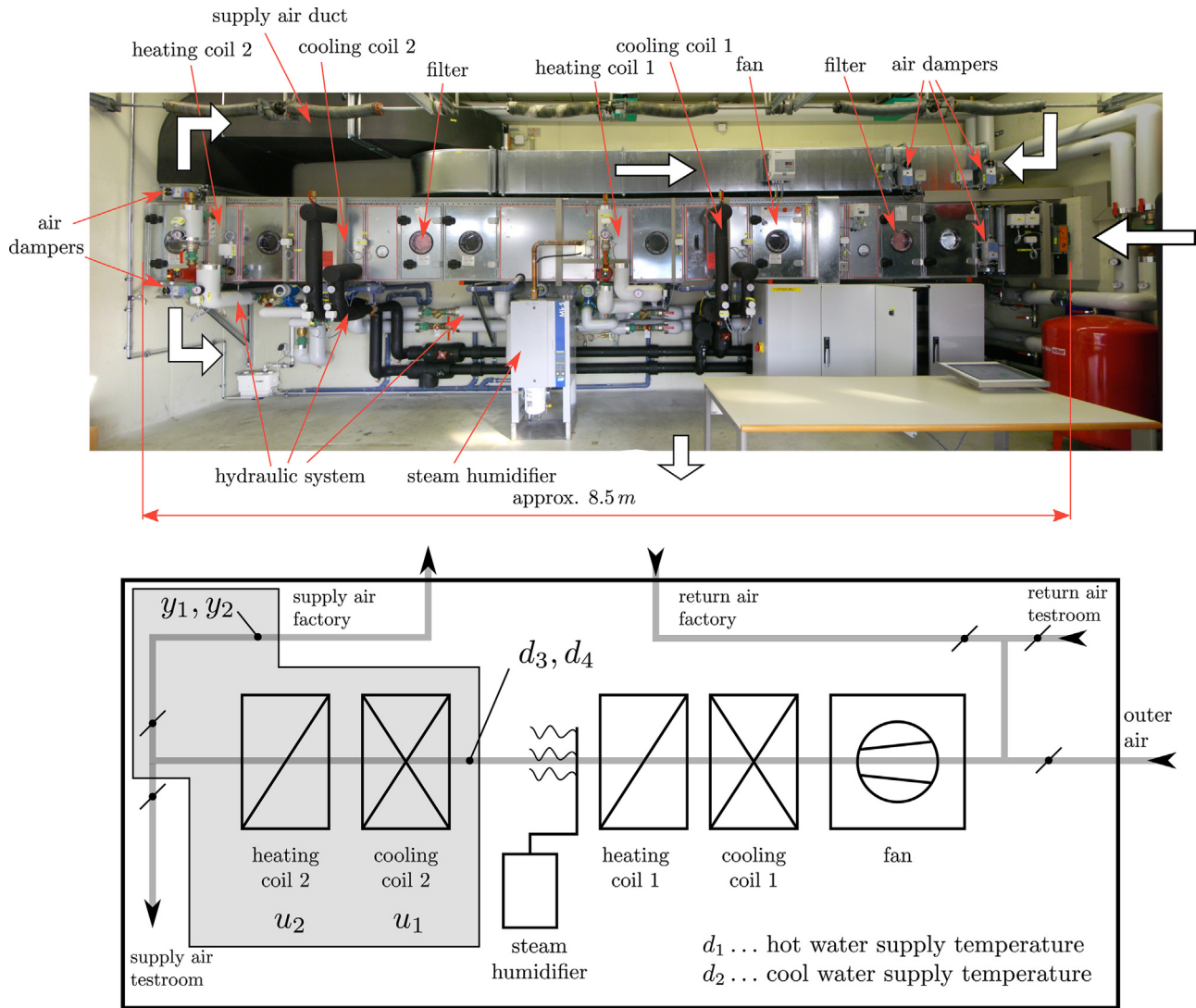


Fig. 1. Photograph and schematic representation of the HVAC test plant.

2.2. Fuzzy modelling

A typical fuzzy model in the T-S form [78] can be given with the rules \mathbf{R}_j , which locally describe the linear input–output relations of the state-space system

\mathbf{R}_j : if $x_{p1}(k)$ is $\mathbf{P}_{j,1}$ and ... and $x_{pq}(k)$ is $\mathbf{P}_{j,q}$
 then $x_m(k + 1) = \mathbf{A}_{m,j}x_m(k) + \mathbf{B}_{m,j}u_m(k - T_{Dm}) + \mathbf{R}_{m,j}(k)$, (1)

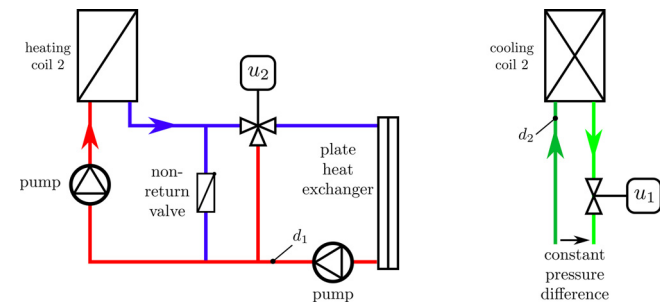


Fig. 2. The water loops of the heating and cooling coil 2.

where k is a time sample, x_m are the system states, u_m are the system inputs, $\mathbf{A}_{m,j}$ is a system matrix, $\mathbf{B}_{m,j}$ is the input matrix and $\mathbf{R}_{m,j}$ is the residual vector of the fuzzy model for every rule, and T_{Dm} is the time delay, which can be estimated from the system responses. The q -element vector $x_p^T(k) = [x_{p1}(k), \dots, x_{pq}(k)]$ represents the premise of the fuzzy system, and $\mathbf{P}_{j,q}$ are the fuzzy sets, where $j = 1, \dots, n_f$ represents the number of all fuzzy rules. Also r -element vector $x_c^T(k) = [x_{c1}(k), \dots, x_{cr}(k)]$, usually referred to as consequence vector, represents the input to the fuzzy system.

The model output can be determined based on the fuzzy rules and the associated values of the membership function. The whole of the discrete system output with the use of T-norm [79] is as follows

$$x_m(k + 1) = \frac{\sum_{j=1}^{n_f} \prod_{i=1}^q \mu_{P_{j,i}}(x_{pi}) \phi(x_m, u_m, \mathbf{R}_{m,j}, k)}{\sum_{j=1}^{n_f} \prod_{i=1}^q \mu_{P_{j,i}}(x_{pi})}, \quad (2)$$

$$\phi(x_m, u_m, \mathbf{R}_{m,j}, k) = \mathbf{A}_{m,j}x_m(k) + \mathbf{B}_{m,j}u_m(k - T_{Dm}) + \mathbf{R}_{m,j}(k),$$

where $\phi(x_m, u_m, \mathbf{R}_{m,j}, k)$ is a linear function and $\mu_{P_{j,i}}(x_{pi})$ is a real function that produces the membership grade of the variable x_{pi} ($i = 1,$

... , q) with respect to the fuzzy set $\mathbf{P}_{j,i}$. Eq. (2) can be simplified with a partition of unity where the following functions

$$\beta_j(x_p) = \frac{\prod_{i=1}^q \mu_{P_{j,i}}(x_{pi})}{\sum_{j=1}^{n_f} \prod_{i=1}^q \mu_{P_{j,i}}(x_{pi})} \quad (3)$$

give information about the fulfillment of the respective fuzzy rule in the normalized form, where it is obvious that the sum of $\beta_j(x_p)$ equals 1. By combining (2) and (3) the discrete fuzzy state-space model can be rewritten as a combination of the input and residual vector responses of the system

$$\begin{aligned} x_m(k+1) &= \sum_{j=1}^{n_f} \beta_j(x_p) [\mathbf{A}_{m,j}x_m(k) \\ &+ \mathbf{B}_{m,j}u_m(k - T_{D_m}) + \mathbf{R}_{m,j}(k)], \\ y_m(k) &= \sum_{j=1}^{n_f} \beta_j(x_p)\mathbf{C}_{m,j}x_m(k), \end{aligned} \quad (4)$$

where $\mathbf{C}_{m,j}$ is the output matrix of the fuzzy model for every rule and $y_m(k)$ is the output of the fuzzy model.

The approach requires a reliable input-output data set to build the T–S fuzzy model. The best way to obtain this reliable input–output data set is to make various open-loop experiments on a real plant, of course if these kinds of experiments can be done on a real process. The other possibility would be to build a detailed mathematical model of a real plant first, but that requires a lot of effort. With the obtained mathematical model the simulation experiments can be carried out without any concerns and the limitations that we can expect with real-world experiments. In the case that the real-world system can be excited properly, it can be the single source of information for generating the input–output data used to obtain the required process model. When there are no disturbances acting on the systems, this is typically possible and the real-world measurements are the method of choice. In the case that the disturbances acting on the real world system during the identification experiments cover the required range/dynamics, the use of the real-world signals is also possible. However, if this is not the case (e.g., the disturbances are prescribed and they do not excite the system properly during the identification experiments), a mathematical model is required to generate the data.

We built the T–S fuzzy model from the input–output identification data (with a sampling time of 1 s) obtained from the detailed HVAC mathematical model [11,12] using the already-mentioned Gustafson–Kessel clustering method [55], and with the help of the Fuzzy Identification Toolbox for the Matlab environment [80] the fuzzy parameters of the local linear models in 8 fuzzy clusters for each output were obtained.

We used the smooth (Gaussian) membership functions with the “projected” option provided in the mentioned toolbox. Due to the finite length of the identification signal and the finite number of different input values, not all operating points can be met, but we took range 280.15–321.15 K (7–48 °C) for y_1 and 8–100% for y_2 into account. We obtained the T–S fuzzy model of the structure $y_m(k+1) = f(y_m(k), u_m(k))$, which was then transformed to the state space form. The antecedent vector consisted of eight elements, the process outputs, the process inputs and four measurable disturbances (or external process inputs) in each step, $x_p^T(k) = [y_1(k), y_2(k), u_1(k), u_2(k), d_1(k), d_2(k), d_3(k), d_4(k)]$. We obtained the model parameters for both outputs that multiply the associated elements of the consequent vector $x_c^T(k) = [y_1(k), y_2(k), u_1(k), u_2(k), d_1(k), d_2(k), d_3(k), d_4(k), r_1(k)]$. Our consequent vectors dimensions are for one greater than antecedent vectors dimension due to proposed controller structure and the need for the same number of system inputs and outputs due to the inverse matrix calculation.

Table 1
Fuzzy model parameters.

Symbol	Description	Unit
$y_1(k)$	First controlled variable, the air temperature	K (°C)
$y_2(k)$	Second controlled variable, the air relative humidity	%
$u_1(k)$	First actuator, the valve opening for the cooling coil 2	%
$u_2(k)$	Second actuator, the valve opening for the heating coil 2	%
$d_1(k)$	First measurable disturbance, the hot water supply temperature for the heating coil hydraulic circuit	K (°C)
$d_2(k)$	Second measurable disturbance, the cool water supply temperature for the cooling coil hydraulic circuit	K (°C)
$d_3(k)$	Third measurable disturbance, the air temperature at the inlet of cooling coil 2	K (°C)
$d_4(k)$	Fourth measurable disturbance, the air relative humidity at the inlet of cooling coil 2	%
$r_f(k)$	Residual of the fuzzy model	/

The symbols, descriptions and units of the fuzzy model parameters are gathered in Table 1.

There are many fuzzy models of various forms and structures, also various fuzzy identification methods are being used, but we have chosen the described ones due to their simplicity. In Fig. 3 are the results of the model identification, from which we calculated the root-mean-square (RMS) error values of 2.3 K and 6.7% for the two signals, respectively, and in Fig. 5 are the results of the model validation, where the RMS error values were 2.3 K and 8.9% for the two signals, respectively. In both figures the solid blue line is the process and the dashed red line is the model signal. The validation input-output data was also obtained from the mentioned HVAC mathematical model. In Figs. 4 and 6 the measured disturbances from the identification and the validation are presented. We consider the mixed-air dry-bulb temperature and the relative humidity—these are the disturbances d_3 and d_4 . For our experiments, the knowledge of these “mixed-air properties” is sufficient, since the mixed air is the “input” to our cooling coil, i.e., to our first actuator. The mixed-air properties of course depend on the outer air and the return air, but a detailed knowledge of them is not required in our experiments, because the mixed-air dry-bulb temperature and the relative humidity are measured directly.

Figs. 5 and 6

For the identification and validation we used a lot of data because we could generate it using a mathematical model simulation. A lot of data normally means a more accurate fuzzy model, but the data from the real-world plant would probably be shorter. There is no problem with building the fuzzy model from lesser

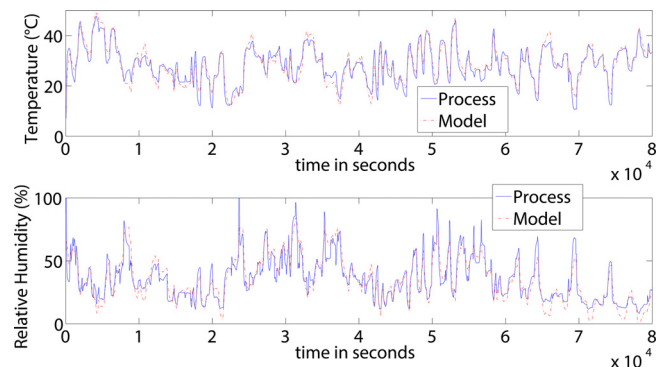


Fig. 3. Identification of the fuzzy model outputs.

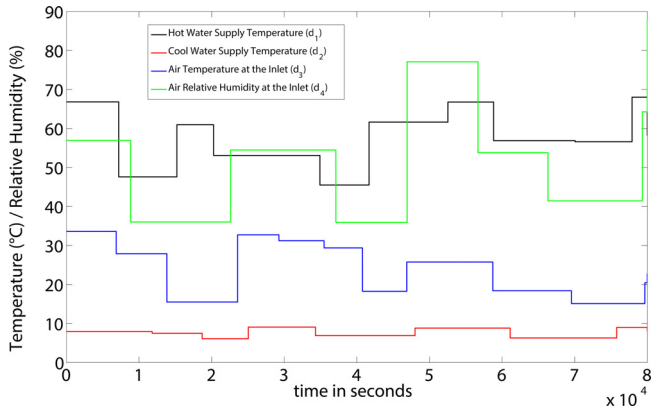


Fig. 4. Four measurable disturbances by the identification.

data, although the fuzzy model might be less accurate then. The proposed approach in Section 3 is very robust to the real process-fuzzy model differences, so we could expect similar closed-loop performance also with a less accurate fuzzy model. The duration of approximately one day for the excitation experiment proved to be a good compromise with respect to the model accuracy and the required identification signal length.

3. Fuzzy model-based multivariable predictive functional control

3.1. Control algorithm

Here we present the extension of the basic principles of the predictive functional control, which are very solid, natural and easy to

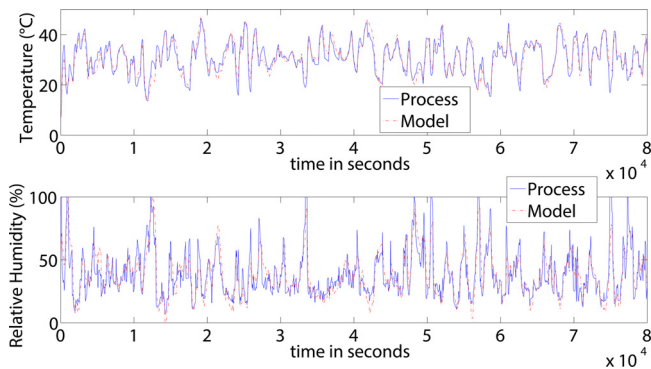


Fig. 5. Validation of the fuzzy model outputs.

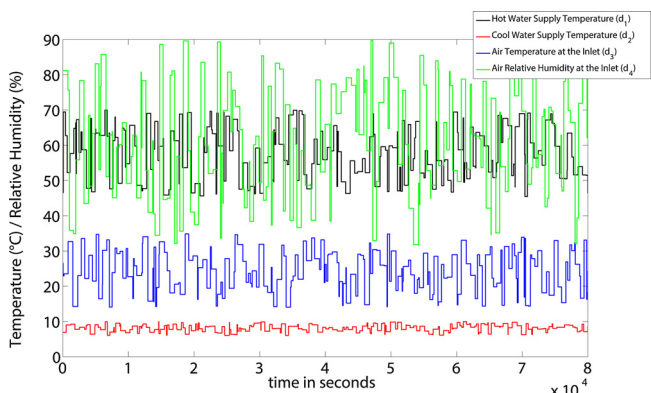


Fig. 6. Four measurable disturbances by the validation.

understand. The graphical representation of the major steps for the proposed approach design are presented in Fig. 7.

In step 1 a Fuzzy Model is developed—in substep 1 the Input/Output data are measured, then in substep 2 the Fuzzy Identification is performed and in substep 3 the Fuzzy Model is validated in verified. If the Fuzzy Model is not as expected, we must repeat substep 1. In step 2 the FMBMPC Controller is designed—in substep 4 the Control demands are set and in substep 5 the Controller Parameters are calculated and adjusted (if needed). If the Controller is not feasible, we must go back to substep 4. In step 3 in substep 6 the closed-loop simulation tests are carried out, and if the simulated system performance is bad, we have to repeat substep 5. In step 4 in substep 7 the Code is generated and transferred to the PLC, and in substep 8 the closed-loop real-world tests are carried out. If the real-world system performance is not good, we have to go back to substep 6. In the end in substep 9 the results are evaluated and compared.

The basis of the predictive control algorithm in general is the explicit use of a dynamic process model to predict the future process-output behaviour over a finite horizon and to evaluate the control in such a way so as to minimize the chosen cost function. The predictive control law is in general obtained by minimizing of the following criterion

$$J(u, k) = \sum_{l=N_1}^{N_2} (y_m(k+l) - y_r(k+l))^2 + \lambda \sum_{l=N_1}^{N_u} u^2(k+l), \quad (5)$$

where $y_m(k+l)$, $y_r(k+l)$ and $u(k+l)$ stand for the l -step-ahead prediction of the model output signal, the reference trajectory and the control signal. N_1 , N_2 and N_u are the minimum, maximum and control horizon, and λ weights the relative importance of the control and output variables. For each time step the optimal control sequence according to the criterion is obtained, but only the first element is used and applied. The goal of the control is to determine the future control action so that the predicted output trajectory coincides with the reference trajectory that is given in the form of the reference model. Only one coincidence point (horizon) is assumed ($N_1 = N_2 = N_u$), which is called the coincidence horizon (H). Instead of minimizing the criterion function it is assumed that at the coincidence horizon the predicted output value coincides with the reference trajectory. The prediction is calculated using the known strategy of mean-level control [81] under the assumption of constant future manipulated variables $u(k) = u(k+1) = \dots = u(k+H-1)$. The main idea of the PFC is the equivalence of the objective increment of the process Δ_p and the model output increment Δ_m .

The problem of the delays in the plant is circumvented by constructing an auxiliary variable that serves as the output of the plant if there are no delays present. A discrete, non-delayed stable process model must be given in the state-space form with the matrices $\mathbf{A}_{m,j}$, $\mathbf{B}_{m,j}$, $\mathbf{C}_{m,j}$, and $\mathbf{R}_{m,j}$

$$x_m^0(k+1) = \sum_j \beta_j(x_p) [\mathbf{A}_{m,j} x_m^0(k) + \mathbf{B}_{m,j} u(k) + \mathbf{R}_{m,j}(k)], \quad (6)$$

$$y_m^0(k) = \sum_j \beta_j(x_p) \mathbf{C}_{m,j} x_m^0(k),$$

where x_m^0 is the non-delayed model state vector and y_m^0 is the non-delayed model output. The stability of the system can be guaranteed with the matrix $\mathbf{A}_{m,j}$ eigenvalues checking and using one of the criterions for the fuzzy systems stability, for example LMI used in [82]. The frozen-time theory [83,84] makes it possible to

find a relation between the nonlinear dynamical system and the associated linear time-varying system

$$\begin{aligned} x_m^0(k+1) &= \bar{\mathbf{A}}_m x_m^0(k) + \bar{\mathbf{B}}_m u(k) + \bar{\mathbf{R}}_m(k), \\ y_m^0(k) &= \bar{\mathbf{C}}_m x_m^0(k), \end{aligned} \tag{7}$$

where we can write $\bar{\mathbf{A}}_m = \sum_j \beta_j(x_p(k)) \mathbf{A}_{m,j}$, $\bar{\mathbf{B}}_m = \sum_j \beta_j(x_p(k)) \mathbf{B}_{m,j}$, $\bar{\mathbf{C}}_m = \sum_j \beta_j(x_p(k)) \mathbf{C}_{m,j}$, and $\bar{\mathbf{R}}_m = \sum_j \beta_j(x_p(k)) \mathbf{R}_{m,j}$.

Based on the above-mentioned assumptions and with the help of Eq. (7) we can obtain an H -step-ahead prediction of the non-delayed model

$$\begin{aligned} y_m^0(k+H) &= \bar{\mathbf{C}}_m \left[\bar{\mathbf{A}}_m^H x_m^0(k) \right. \\ &\left. + \left(\bar{\mathbf{A}}_m^H - \mathbf{I} \right) \left(\bar{\mathbf{A}}_m - \mathbf{I} \right)^{-1} \left(\bar{\mathbf{B}}_m u(k) + \bar{\mathbf{R}}_m \right) \right], \end{aligned} \tag{8}$$

where \mathbf{I} is an identity matrix. The reference model is given in the form

$$\begin{aligned} x_r(k+1) &= \mathbf{A}_r x_r(k) + \mathbf{B}_r w(k), \\ y_r(k) &= \mathbf{C}_r x_r(k), \end{aligned} \tag{9}$$

where x_r is the reference model state vector, w is the reference signal and y_r is the reference model output. The matrices \mathbf{A}_r (system matrix), \mathbf{B}_r (input matrix) and \mathbf{C}_r (output matrix) of the reference model must satisfy the equation

$$\mathbf{C}_r (\mathbf{I} - \mathbf{A}_r)^{-1} \mathbf{B}_r = \mathbf{I}, \tag{10}$$

which enables reference-trajectory tracking due to the unity gain for each channel. We can use a first-order reference model, and so its matrices become diagonal. Furthermore, we can choose $\mathbf{C}_r = \mathbf{I}$, which must lead to $\mathbf{B}_r = \mathbf{I} - \mathbf{A}_r$. The reference model prediction using Eqs. (9) and (10) can then be given as

$$y_r(k+H) = \mathbf{A}_r^H y_r(k) + \left(\mathbf{I} - \mathbf{A}_r^H \right) w(k), \tag{11}$$

where a constant and bounded reference signal $w(k+i) = w(k)$, $i = 1, \dots, H$ is assumed. If we take into account the main idea of the PFC approach and the reference-trajectory tracking $y_r(k+i) = y_p^0(k+i)$, $i = 1, \dots, H$, where y_p^0 represents the nondelayed process output, we can rewrite (11) as

$$w(k+H) - y_r(k+H) = \mathbf{A}_r^H (w(k) - y_r(k)), \tag{12}$$

and as a result we obtain

$$y_p^0(k+H) = w(k+H) - \mathbf{A}_r^H (w(k) - y_p^0(k)). \tag{13}$$

The objective increment of the process that follows from Eq. (13) is defined as

$$\begin{aligned} \Delta_p &= y_r(k+H) - y_p^0(k) \\ &= w(k+H) - \mathbf{A}_r^H (w(k) - y_p^0(k)) - y_p^0(k). \end{aligned} \tag{14}$$

The non-delayed process output is not directly measurable, so we estimate it with the available signals

$$y_p^0(k) = y_p(k) - y_m(k) + y_m^0(k), \tag{15}$$

where y_p represents the process output. After that the objective increment of the model is defined as

$$\Delta_m = y_m^0(k+H) - y_m^0(k). \tag{16}$$

Next, we equate both the objective increments from Eqs. (14) and (16), and introduce Eq. (8). With some additional calculations we obtain

$$\begin{aligned} \mathbf{G}_0 u(k) &= \left(\mathbf{I} - \mathbf{A}_r^H \right) \left(w(k) - y_p^0(k) \right) \\ &- \mathbf{G}_R - \bar{\mathbf{C}}_m \bar{\mathbf{A}}_m^H x_m^0(k) + y_m^0(k), \end{aligned} \tag{17}$$

where the new matrices are $\mathbf{G}_0 = \bar{\mathbf{C}}_m \left(\bar{\mathbf{A}}_m^H - \mathbf{I} \right) \left(\bar{\mathbf{A}}_m - \mathbf{I} \right)^{-1} \bar{\mathbf{B}}_m$ and $\mathbf{G}_R = \bar{\mathbf{C}}_m \left(\bar{\mathbf{A}}_m^H - \mathbf{I} \right) \left(\bar{\mathbf{A}}_m - \mathbf{I} \right)^{-1} \bar{\mathbf{R}}_m$.

Introducing Eq. (15) into (17) we finally obtain the control law of the FMBMPC algorithm in an analytical form

$$\begin{aligned} u(k) &= \mathbf{G}_0^{-1} \\ &\times \left[\left(\mathbf{I} - \mathbf{A}_r^H \right) \left(w(k) - y_p(k) + y_m(k) \right) \right. \\ &\left. + \left(\mathbf{A}_r^H \bar{\mathbf{C}}_m - \bar{\mathbf{C}}_m \bar{\mathbf{A}}_m^H \right) x_m^0(k) - \mathbf{G}_R \right]. \end{aligned} \tag{18}$$

Note that the control law Eq. (18) is realizable if the matrix \mathbf{G}_0 is non-singular. Scheme of the FMBMPC controller is presented in the Fig. 8.

3.2. Controller tuning rules

The advantage of the predictive functional controller compared to the alternatives is the easy tuning due to there being only a few controller parameters that need to be set. The first requirement is, of course, to have the fuzzy plant model written in the non-delayed discrete state-space form with the matrices $\mathbf{A}_{m,j}$, $\mathbf{B}_{m,j}$, $\mathbf{C}_{m,j}$ and $\mathbf{R}_{m,j}$, which can be obtained from the fuzzy model output parameters. From here on there are just two controller parameters to tune: the discrete reference model matrix \mathbf{A}_r and the coincidence horizon H . We investigated the FMBMPC controller's behaviour and performance using the different parameter settings on various simple and complex models.

The discrete reference model matrix \mathbf{A}_r can be presented as a diagonal matrix of the reference model's discrete time constants

$$\mathbf{A}_r = \begin{bmatrix} a_{r1} & 0 & \dots & 0 \\ 0 & a_{r2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & a_{rn} \end{bmatrix}, \tag{19}$$

where n stands for the number of model states and a_{rn} are the discrete time constants of each reference model output. Following that we can present the tuning rules for the FMBMPC parameters like

$$T_{rn} = T_{mn}/3, \tag{20}$$

$$H \geq \rho,$$

where T_{rn} is the continuous time representation of a_{rn} given in seconds and T_{mn} is the continuous time constant of each plant model output. The horizon H is given as an integer in the time samples. Note that the relative degree ρ is the difference between the number of poles and the number of zeroes of the system. $H \geq \rho$ is only valid for stable open-loop systems with all open-loop transmission zeroes inside the unit circle. In other cases the setting $H > \rho$ must be used. The rules in Eq. (20) are similar to the rules used by the linear multivariable predictive approach [58], but here we tightened them slightly and in this form they can also hold for the linear multivariable approach. The presented tuning rules in (20) were obtained on the basis of FMBMPC approach performance testing on various simple and complex models.

The presented tuning rules are, in general, some guidelines, which normally give good results, if we consider the trade-off

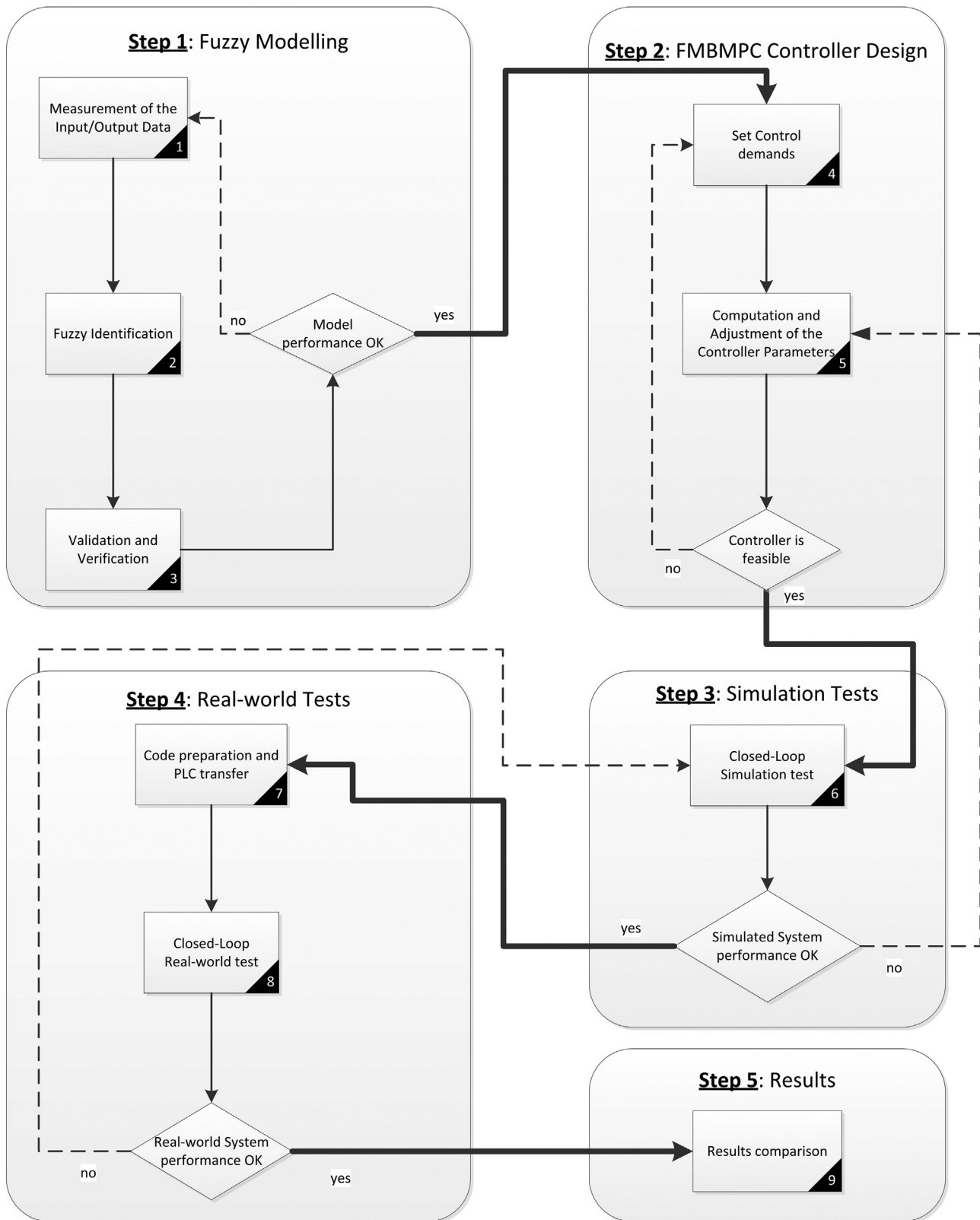


Fig. 7. Illustration of the major steps for the proposed approach design.

between the robustness and the performance of the controlled system. Of course, the control requirements can be different, so we can also set the controller parameters either higher or lower. However, we must be careful because a lower T_{rn} means a tighter control loop, and so the closed loop can become unstable. A lower H also means

a faster control loop and so a lot of noise can be propagated through the closed-loop system.

The stability issues of the proposed approach are here not addressed, since the main idea is the real-time PLC implementation and very accurate and energy-efficient real-world results.

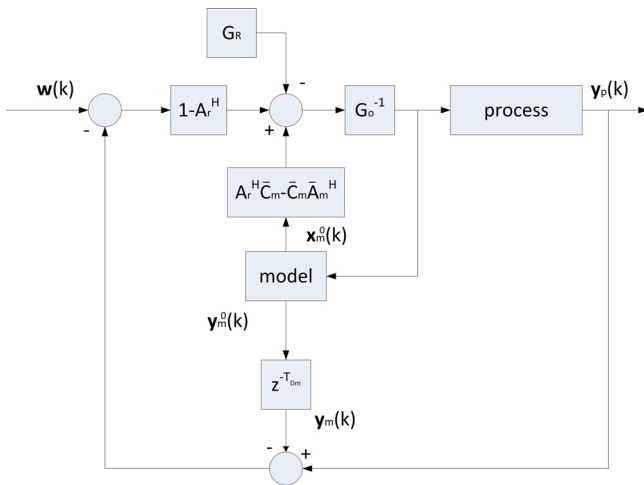


Fig. 8. FBMPC controller scheme.

We are aware of the stability issues and will address them in our future work. Here we examined the previous findings [66], where the stability of the control algorithm regarding the parameter H was studied for the fuzzy SISO approach, and which can also be extended for the fuzzy MIMO approach. If H is less than the maximum relative degree ρ of the plant model, the matrix G_0 becomes singular and the control law is not applicable. When H is equal to ρ , the obtained closed-loop control is only stable if all the open-loop transmission zeroes are inside the unit circle. And when H tends to infinity the system matrix of the closed-loop system goes to the model system matrix, from which we can conclude that a stable control law can always be obtained for open-loop stable systems, even if some open-loop transmission zeroes are outside the unit circle when a suitable H is used. The details of these conditions can be found in [66], which are also true for the presented approach.

4. Implementation

4.1. Fuzzy predictive approach

The fuzzy modelling, the fuzzy model, its parameters and the results are already presented in Subsection 2.2. A sampling time (T_s) of 1 s was used. The FBMPC controller scheme was designed in the Matlab/Simulink environment with code-generation compliance, from which we were able to produce the C code for a real-world programmable logic controller (PLC).

From the detailed HVAC system mathematical model output data we estimated the first-order approximations of the time constants, which we can write as $T_{mc} = 190$ s for cooler to temperature, and $T_{mh} = 150$ s for heater to relative humidity. The same way we estimated the delays, where the direct branch delays accounted for were $D_{mc} = 60$ s and $D_{mh} = 7$ s, for the cooler and heater respectively. With FBMPC we try to nullify the multi-variable interactions, so the reference models' system matrix has only diagonal elements different than zero. The cross-correlated delays are not needed, as the discrete time constants of the cross-correlations in Eq. (21) are zero.

In Eq. (20) we presented some general tuning rules for the fuzzy predictive approach, which hold perfectly for the relative humidity, but for the temperature we had to use slightly faster rules, due to the control performance demands. During the real-world experiments we found that the presented tuning rules give satisfactory responses for the relative humidity, but slightly too slow responses for the temperature. Presented in numbers, $T_{mc}/3$ would be 63.3 s,

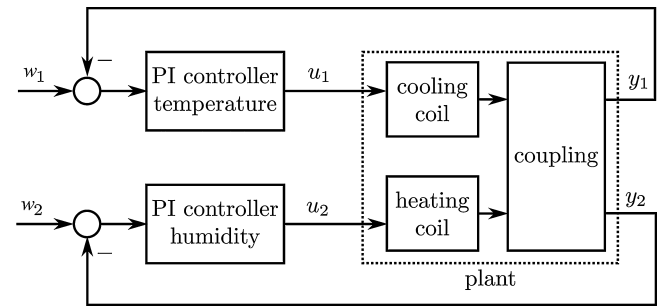


Fig. 9. PI control strategy for the HVAC plant.

but we had to use $T_{r1} = 50$ s, and $T_{r2} = T_{mh}/3$ is 50 s. In terms of Eq. (19) this means

$$A_r = \begin{bmatrix} 0.9802 & 0 \\ 0 & 0.9802 \end{bmatrix}. \tag{21}$$

For the horizon we determined $H = 10$, and so not too much noise is propagated through the closed-loop system (the minimum regarding (20) would be $H = 1$).

4.2. Classical PI approach

The proposed concept is compared against a PI control strategy that is widely used to control HVAC systems in industrial practice [1,3,85]. Two separate PI controllers are designed, one for temperature and one for relative humidity control. The coupling of the controlled variables is neglected in this approach. The temperature control is performed with the help of the cooling coil, and relative humidity control with the help of the heating coil. Of course, the humidity ratio expressed in kg of water per kg of dry air cannot be altered via the heating coil. In the presented setup, the relative air humidity expressed in percent represents the controlled variable y_2 . Since the relative air humidity is a function of the air-humidity ratio and the dry-bulb temperature, y_2 can be influenced indirectly by adjusting the temperature, which is done via the heating coil. This—initially wrong—assignment between the actuators and the controlled variables proved to be a well-suited architecture in practice when multiple PI controllers are used to control the dry-bulb temperature and the relative air humidity. Fig. 9 shows the structure of the control loop.

The parameters of the PI controller were set empirically by a company specialized for controller tuning in HVAC applications. However, systematic approaches based on describing the function analysis to avoid limit cycles can be found in [86]. The PI controllers were implemented on the PLC including an anti-windup strategy. The parameters proportional gain $K_{p,T}$ and $K_{p,Hr}$, and the reset time $T_{i,T}$ and $T_{i,Hr}$ of the controllers were set to

$$\begin{aligned} K_{p,T} &= 12, & T_{i,T} &= 200 \text{ s}, \\ K_{p,Hr} &= 2, & T_{i,Hr} &= 450 \text{ s}, \end{aligned} \tag{22}$$

where the subscripts T and Hr denote the PI controller for temperature and for relative humidity, respectively.

5. Results and discussion

Using the controller settings from the previous section we obtained the following results in Figs. 10 and 11 on a temperature–relative humidity profile lasting almost 9000 s with the following steps:

- first, the temperature was raised from 293.15 K (20 °C) to 302.15 K (29 °C),

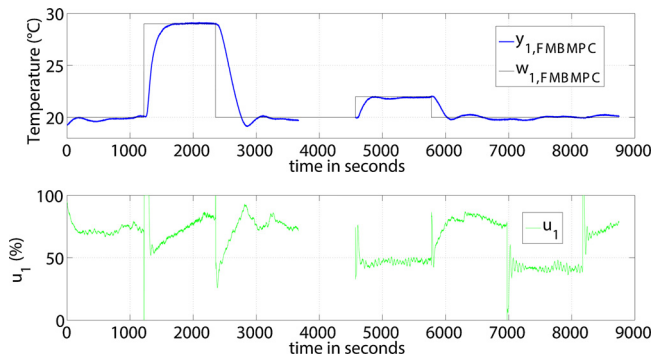


Fig. 10. The system and the controller temperature output.

- and right after that the relative humidity was lowered from 60% to 35%,
- next, the temperature was lowered back to 293.15 K (20 °C), and the relative humidity was raised back to 60%,
- after that the temperature was raised to 295.15 K (22 °C) and lowered back to the initial 293.15 K (20 °C), while the relative humidity stayed constant.
- Finally, the relative humidity was raised to 70%, and again lowered back to the initial 60%, with constant temperature.

The temperature–relative humidity profiles in this section were chosen from the points in the Mollier diagram that were possible to reach.

In all the figures in this section the solid blue line is the signal with the FMBMPC approach, the solid black line is the reference signal, and the solid green line is the FMBMPC approach actuator signal.

In Figs. 10 and 11 we can see only the results with the FMBMPC approach, since we do not have the PI approach measurements on the mentioned temperature–relative humidity profile. Around a time of 4000 s we can see some lack of insignificant data, which is due to lost communication with the PLC. This happened accidentally and is just a coincidence, but it is not the same case as in Figs. 13–16, because the PI response tests were carried out in a separate experiment. From Figs. 10 to 12 it is obvious that the missing data is insignificant. Since the relevant data is available (the transients are completely available to judge the dynamic behaviour, and some parts with constant references are available to judge the fixed set-point control), we decided to take these measurements for presentation in the paper and we did not repeat the measurement. In Fig. 12 the measured disturbances from the first set of the results are presented.

Generally, we obtained very good results, where the overshoot was never more than 1 K for the temperature and always below 5%

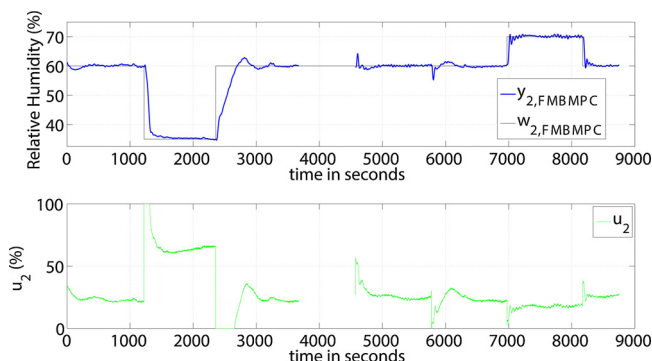


Fig. 11. The system and the controller humidity output.

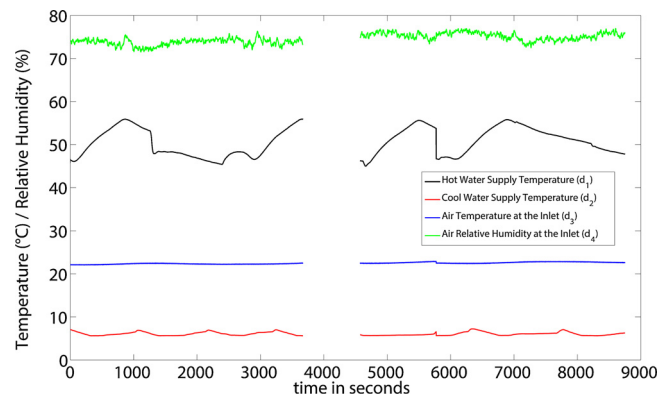


Fig. 12. Four measurable disturbances by the first set of the results.

for the relative humidity. We measured the RMS error value, which was 1.6 K for the temperature, and 3.4% for the relative humidity.

To compare the results to a classical PI approach, additional experiments were performed on a different temperature–relative humidity profile lasting almost 6000 s with the following steps:

- at the same time the temperature was raised from 293.15 K (20 °C) to 302.15 K (29 °C), and the relative humidity was lowered from 50% to 32%,
- next, again at the same time the temperature was lowered back to 293.15 K (20 °C), and the relative humidity was raised back to the initial 50%,
- after that the relative humidity remained constant,
- but the temperature was again raised to 295.15 K (22 °C) and then lowered back to the initial 293.15 K (20 °C).

In the next figures the dashed red line is the signal with the classical PI approach, and the dashed magenta line is the PI approach actuator signal.

The results are shown in Figs. 13 and 14, where we can see some lack of the FMBMPC approach data around the times 1000 and 3000 s (the same lack of the data can be seen in Figs. 15 and 16), which is due to a faster settling time and that is why we earlier stopped the FMBMPC–approach data storing. First, the FMBMPC response tests were carried out and after that we carried out the PI response tests. The FMBMPC approach has a considerably shorter settling time, so we stopped the data storing earlier. After that we detected a longer settling time using the PI approach, but could not repeat the FMBMPC response tests with longer data store. From Figs. 13 and 14 (and Figs. 15 and 16) it is clear that the FMBMPC responses are settled and that again the missing data is insignificant. The focus of these experiments was to investigate the transient response of the FMBMPC approach compared

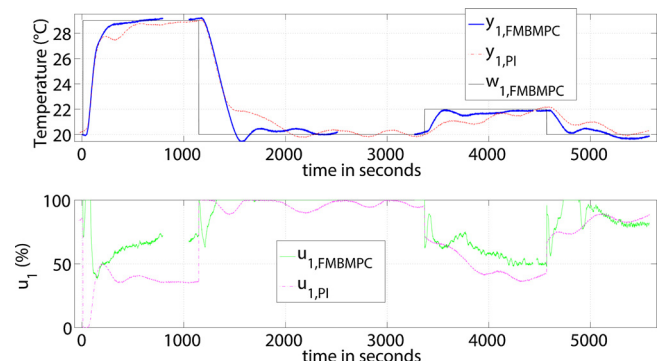


Fig. 13. The system and both controllers' temperature outputs.

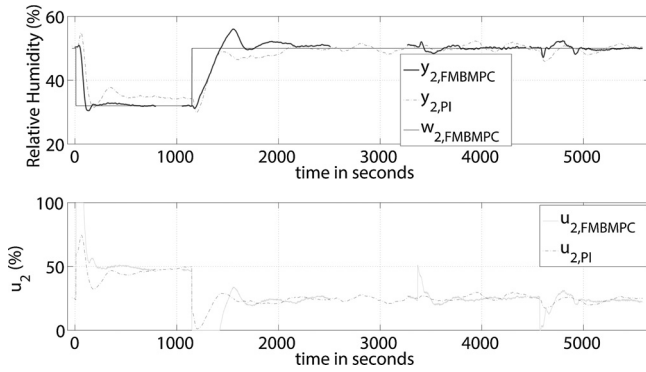


Fig. 14. The system and both controllers' humidity outputs.

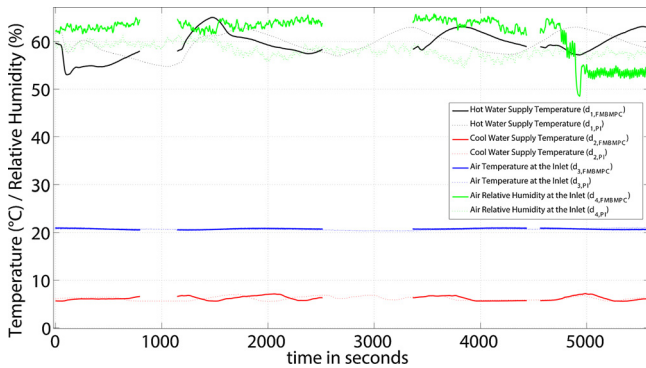


Fig. 15. Four measurable disturbances by the FMBMPC and PI approach experiment.

to the classic PI approach—especially concerning the energy consumption during the transients. Consequently, for our analyses, the considered time spans for the PI and the FMBMPC experiments are sufficient. In Fig. 15 the measured disturbances from the second set of the results are presented, for the FMBMPC approach experiment with solid lines and for the PI approach experiment with dashed lines.

We can indicate the main drawbacks of the classical PI approach, which are its questionable performance at some operating points, a varying performance across the whole operating range, a slower arrival to the exact set point, a poorer performance by the disturbance rejection, and a more oscillating response. In general, the accuracy and the disturbance rejection are significantly worse when using the classical approach. Again, the overshoot by the FMBMPC approach is never more than 1 K for the temperature and always below 6% for the relative humidity.

We also measured and compared the performance using the RMS error value. For the temperature output we obtained the values

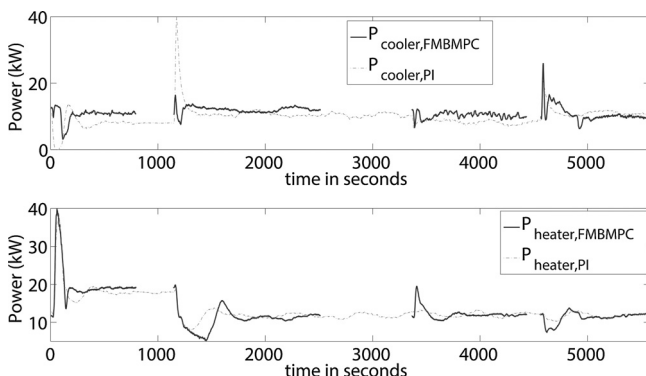


Fig. 16. Both controllers' electrical power consumption.

7.1 K and 7.9 K, and for the relative humidity output the values 12.2% and 15.8% for the FMBMPC and PI approaches, respectively, from which we can also see that the proposed FMBMPC control algorithm outperforms the classical PI approach. In terms of the percentage, that means slightly more than 12% better for the temperature control, and almost 30% better for the relative humidity control.

We investigated and compared also energy consumption. Since the consumption in steady state is very similar for both approaches, we concentrated only on the consumption until steady state for both outputs is reached. We defined the desired range from the set point for the temperature as ± 0.5 K, and for the relative humidity $\pm 3\%$ (by the first step) and $\pm 2\%$ (by all other changes). We calculated the consumed energy as

$$W_c = P_c \cdot t_c = \phi_{mf} \cdot c_w \cdot (\vartheta_{in} - \vartheta_{out}) \cdot t_c, \quad (23)$$

where W_c stands for consumed energy, P_c stands for consumed power, t_c stands for operating time, ϕ_{mf} stands for water mass flow, c_w stands for water specific heat capacity, ϑ_{in} stands for the inlet water temperature and ϑ_{out} stands for the outlet water temperature. In the next figure the solid blue line is the power with the FMBMPC approach and the dashed red line is the power with the classical PI approach.

In Fig. 16 the consumed power for both approaches is presented. The consumed power is presented in kW units (and not in $\text{kg} \cdot \text{m}^2 \cdot \text{s}^{-3}$) due to it being a more natural presentation. In the same time span the FMBMPC approach consumes more energy, but it reaches the desired range before classical PI approach, which is less effective. To reach similar or worse results the control with the PI approach lasts longer and so the energy consumption of the proposed predictive algorithm is lower. Altogether we consume $75,240,000 \text{ kg m}^2 \text{ s}^{-2}$ (20.9 kW h) of energy with the PI approach, and just $42,120,000 \text{ kg m}^2 \text{ s}^{-2}$ (11.7 kW h) of energy with FMBMPC approach. The difference is obvious, with the proposed FMBMPC approach we consume $33,120,000 \text{ kg m}^2 \text{ s}^{-2}$ (9.2 kW h) or 44% less energy.

If we compare the unsteady time period and the steady time period of the responses, the unsteady time period represents a shorter portion of the response. The energy-consumption comparison is highly dependent on the usage type of the HVAC system. Where set point changes are very frequent (for example, in the inlet air conditioning of combustion-engine test benches), the FMBMPC approach would save a lot of energy. Of course, where the set point changes are rare (and the set points are mostly constant), there would not be much difference in the energy-consumption between the presented approaches. For example, in the buildings there can be different temperature–relative humidity profiles for all four seasons, even different for the day and the night. The profiles can be changed depending on the room or office occupancies, or even according to the schedule (for example, some conference rooms are occupied two hours in the morning and three hours in the afternoon). All these examples mean a relatively frequent set points change and in this case the proposed FMBMPC approach would save a considerable amount of energy.

Many HVAC systems are only operated during daytime, and during the night they are switched off or at least the set-points are changed to settings with less energy consumption during the night. With the proposed approach, the time required to switch on the system (or to change to the daytime set-point) requires the mentioned 44% less energy, as the measurement shows for some exemplary set-point changes. The overall energy efficiency improvement considering the whole operating time of the system is of course much smaller than the 44% savings during switch on/set-point change. An overall energy-efficiency improvement is strongly dependent on the type of operation and therefore we cannot make reliable estimates here. Systems that are only operated

on some days of the week (e.g., the air conditioning of event halls, seminar rooms, etc.) and industrial systems with many set-point changes considerably profit from the proposed method. So, in conclusion, there are many HVAC systems that can profit from the proposed control concept, especially those types of systems mentioned above.

6. Conclusions

We extended our previous researches in the predictive functional control field with a fuzzy model-based multivariable predictive functional approach, which was studied in simulations implemented in a Matlab/Simulink environment on a mathematical model of a HVAC system, and then implemented and tested on a real-world test plant with PLC using code generation and produced C code. The process exhibits a nonlinear and multivariable nature, and in this way the chosen control algorithm matches well with the specialties of the mentioned natures.

The main advantage of the proposed FMBMPC control is in the ease with which it can be understood, its relatively simple design and its high-quality control performance. Of course, the relatively simple design is meant after the obtained fuzzy model and with use of the proposed tuning rules. To obtain the fuzzy model is not a simple task, but it can be simplified with the various tools. We built the fuzzy model using the Gustafson–Kessel clustering method with the help of the Fuzzy Identification Toolbox for the Matlab environment. We also presented general tuning guidelines, which gave good results considering the trade-off between the robustness and the performance. In terms of the percentage, the control performance is 12% better using the temperature control and almost 30% better using the relative humidity control. The consumed-energy comparison showed that using the proposed approach we consume 44% less energy.

After the comparison of the proposed controller approach with classical PI control, we can highlight the main advantages of the FMBMPC algorithm, which are its very good performance across the whole operating range, only a slight variation in the results at all the operating points, the quick rise times and the small overshoots, the superior performance by the disturbance rejection and the great multivariable interactions handling. After the energy consumption comparison we can also say that the FMBMPC approach is not only more effective, but also more energy-efficient.

Our future work will be based on further optimization of the FMBMPC controllers' C code for the PLCs' so that it will be as simple and computational undemanding as possible. We will make further tests on the proposed algorithm using the real-world HVAC plant as well as performance comparisons with other MPC approaches. It would also be very interesting to test the FMBMPC control algorithm on other real-world plants, like batch reactors, heat exchangers, furnaces, pressure vessels, etc., to see if the performance is similarly very good.

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